(Pages : 6) **P – 3856**

Reg. No. : .....

Name : .....

Third Semester B.Sc. Degree Examination, January 2023

First Degree Programme under CBCSS

Mathematics

Complementary Course for Chemistry and Polymer Chemistry

MM 1331.2 – Mathematics III : LINEAR ALGEBRA, PROBABILITY THEORY AND NUMERICAL METHODS

(2019 – 2020 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. Each question carries **1** mark.

1. Define the rank of a matrix.

2. Evaluate the determinant $\begin{vmatrix} 0 & a & -b \\ -a & 0 & c \\ b & -c & 0 \end{vmatrix}$.

3. What is the magnitude of a vector?

4. Define Kronecker δ.

5. What is the scalar product of two vectors?

6. What is the sample space of an event?

7. There are 10 chairs in a row and 8 people to be seated. In how many ways can this be done?

8. Write the expression for variance of a random variable x and explain the terms.

9. Write Baye's formula for conditional probability.

10. What is an algebraic equation?

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions Each question carries **2** marks.

11. Find the rank of the matrix $\begin{pmatrix} 1 & 1 & 2 \\ 2 & 4 & 6 \\ 3 & 2 & 5 \end{pmatrix}$.

12. Evaluate the determinant $\begin{vmatrix} 1 & -5 & 2 \\ 7 & 3 & 4 \\ 2 & 1 & 5 \end{vmatrix}$.

13. Find the cross product of the vectors $A=2i+j-k$ and $B=i+3j-2k$.

14. Find the symmetric equation of the line through $(1,-1,-5)$ and $(2,-3,-3)$.

15. Find the product of A and B if $A=\begin{pmatrix} 4 & 2 \\ -3 & 1 \end{pmatrix}$ and $B=\begin{pmatrix} 1 & 5 & 3 \\ 2 & 7 & -4 \end{pmatrix}$.

16. Define linear functions.

17. Find the probability that a single card drawn from a shuffled deck of cards will be either a diamond or a king.

18. Two dice are rolled. What is the probability that the sum is ≥ 10?

19. Define mutually exclusive events.

20. If $P(A) = 0.07755, P(A\,B) = 0.038$, find $P_A(B)$.

21. What is the probability that a number $n, 1 \leq n \leq 99$. is divisible by both 6 and 10?

22. A club consists of 50 members. In how many ways can a president, vice president, secretary and treasurer be chosen?

23. Write Newton-Raphson iteration formula.

24. Write an iteration scheme for finding the square root of X.

25. What is binary chopping?

26. Evaluate the integral $I=\int_0^1 \frac{1}{1+x^2}\,dx$ using the trapezium rule.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions. Each question carries **4** marks.

27. Write and row reduce the augmented matrix for the equations :

$$x - y + 4z = 5$$
$$2x - 3y + 8z = 4$$
$$x - 2y + 4z = 9$$

28. Evaluate the determinant $D = \begin{vmatrix} 4 & 3 & 0 & 1 \\ 9 & 7 & 2 & 3 \\ 4 & 0 & 2 & 1 \\ 3 & -1 & 4 & 0 \end{vmatrix}$.

29. Using Cramer's rule solve the set of equations :

$$2x + 3y = 3$$
$$x - 2y = 5$$

30. Find the equation of a line through $(1, 0, -2)$ and perpendicular to the plane $3x - 4y + z + 6 = 0$.

31. Find the distance between the lines $r = i - 2j + (i - k)t$ and $r = 2j - k + (j - i)t$.

32. Which is the most probable sum in a toss of two dice? what is its probability?

33. Two students are working separately on the same problem. If the first student has probability $\frac{1}{2}$ of solving it and the second student has probability $\frac{3}{4}$ of solving it. what is the probability that atleast one of them solves it.?

34. Find the coefficient of x^8 in the binomial expansion of $(1 + x)^{15}$.

35. Using Baye's formula find the probability of all heads in three tosses of a coin if you know that atleast one is a head?

36. Evaluate $I = \int_0^2 (x^2 - 3x + 4)\, dx$ using trapezium rule with $h = 0.5$.

37. Evaluate $I = \int_0^1 \frac{1}{1+x^2}\, dx$ using Gaussian integration.

38. Find an explicit formula that will generate a random number y distributed on $(-\infty, \infty)$ according to the Cauchy distribution $f(y)\, dy = \left(\frac{a}{\pi}\right) \frac{dy}{a^2 + y^2}$, given a random number ξ uniformly distributed on (0, 1).

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions. Each question carries **15** marks.

39. Diagonalize $H = \begin{pmatrix} 2 & 3-i \\ 3+i & -1 \end{pmatrix}$.

40. Find the rotation matrix C if the quadratic surface $x^2 + 6xy - 2y^2 - 2yz + z^2 = 24$ is rotated to principal axis.

41. A preliminary test is customarily given to the students at the beginning of a certain course.

The following data are accumulated after several years :

(a) 95% of the students pass the course,

(b) 96% of the students who pass the course also passed the preliminary test.

(c) 25% of the students who fail the course passed the preliminary test.

What is the probability that a student who failed the preliminary test will pass the course?

42. Derive the Poison density function $P_n = \frac{\mu^n}{n!} e^{-\mu}$.

43. Solve the simultaneous equations

$$x_1 + 6x_2 - 4x_3 = 8$$
$$3x_1 - 20x_2 + x_3 = 12$$
$$-x_1 + 3x_2 + 5x_3 = 3$$

using Gaussian elimination.

44. Explain any three Monte Carlo methods.

(2 × 15 = 30 Marks)
